

# Transverse electric conductivity and dielectric permeability in quantum non-degenerate and Maxwellian collisional plasma with variable collision frequency in Mermin's approach

A. V. Latyshev<sup>1</sup> and A. A. Yushmanov<sup>2</sup>

*Faculty of Physics and Mathematics,  
Moscow State Regional University, 105005,  
Moscow, Radio str., 10-A*

## Abstract

Formulas for transverse conductance and dielectric permeability in quantum non-degenerate and Maxwellian collisional plasma with arbitrary variable collision frequency in Mermin's approach are deduced. Frequency of collisions of particles depends arbitrarily on a wave vector. The special case of frequency of collisions proportional to the module of a wave vector is considered. The graphic analysis of the real and imaginary parts of dielectric function is made.

**Key words:** Klimontovich, Silin, Lindhard, Mermin, quantum collisional plasma, conductance, non-degenerate plasma, Maxwellian plasma.

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## 1. Introduction

In Klimontovich and Silin's work [1] expression for longitudinal and transverse dielectric permeability of quantum collisionless plasmas has been received.

Then in Lindhard's work [2] expressions has been received also for the same characteristics of quantum collisionless plasma.

By Kliewer and Fuchs [3] it has been shown, that direct generalisation of formulas of Lindhard on a case of collisionless plasmas, is incorrectly. This lack for the longitudinal dielectric permeability has been eliminated in work of Mermin [4] for collisional plasmas. In this work of Mermin [4] on the basis of the analysis of a nonequilibrium matrix density in  $\tau$ -approach expression for longitudinal dielectric permeability of quantum collisional

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<sup>1</sup>avlatyshev@mail.ru

<sup>2</sup>yushmanov@inbox.ru

plasmas in case of constant frequency of collisions of particles of plasma has been announced.

For collisional plasmas correct formulas longitudinal and transverse electric conductivity and dielectric permeability are received accordingly in works [5] and [6]. In these works kinetic Wigner—Vlasov—Boltzmann equation in relaxation approximation in coordinate space was used.

In work [7] the formula for the transverse electric conductivity of quantum collisional plasmas with use of the kinetic Shrödinger—Boltzmann equation in Mermin’s approach (in space of momentum) has been deduced.

In work [8] the formula for the longitudinal dielectric permeability of quantum collisional plasmas with use of the kinetic Shrödinger—Boltzmann equation in approach of Mermin (in space of momentum) with any variable frequency of collisions depending from wave vector has been deduced.

In our work [9] formulas for longitudinal and transverse electric conductivity in the classical collisional gaseous (maxwellian) plasma with frequency of collisions of plasma particles proportional to the module particles velocity have been deduced.

Research of skin-effect in classical collisional gas plasma with frequency of collisions proportional to the module particles velocity has been carried out in work [10].

In our works [11] and [12] dielectric permeability in quantum collisional plasma with frequency of collisions proportional to the module of a wave vector has been investigated. The case of degenerate plasmas was studied in work [11]. The case of non-degenerate and maxwellian plasmas has been investigated in work [12].

Our work [13] is devoted to transverse conductivity and permeability in quantum collisional plasma with variable frequency of collisions. In the same work the case вырожденной plasmas is considered.

Let’s notice, that interest to research of the phenomena in quantum plasma grows in last years [14] – [27].

In the present work formulas for transverse conductivity and dielectric permeability in quantum non-degenerate and maxwellian collisional plasma with arbitrary variable collision frequency in Mermin’s approach are deduced. Frequency of collisions of particles depends arbitrarily on a wave vector. This work is continuation of our article [13]. The special case

of frequency of collisions proportional to the module of a wave vector is considered. The graphic analysis of the real and imaginary parts of dielectric function is made.

## 2. Transversal electric conductivity and dielectric permeability

In work [13] we receive the following expression of an invariant form for the transversal electric conductivity

$$\sigma_{tr}(\mathbf{q}, \omega, \bar{\nu}) = \frac{ie^2 N}{m\omega} \left[ 1 + \frac{\hbar^2}{8\pi^3 m N} \int \Xi(\mathbf{k}, \mathbf{k} - \mathbf{q})(f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}}) \mathbf{k}_{\perp}^2 d\mathbf{k} \right]. \quad (2.1)$$

Here

$$\Xi(\mathbf{k}, \mathbf{k} - \mathbf{q}) = \frac{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}} - i\hbar\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})}{(\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}})\{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}} - \hbar[\omega + i\hbar\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})]\}},$$

$$\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q}) = \frac{\nu(\mathbf{k}) + \nu(\mathbf{k} - \mathbf{q})}{2}.$$

Let's take advantage of definition of transversal dielectric permeability

$$\varepsilon_{tr}(\mathbf{q}, \omega, \nu) = 1 + \frac{4\pi i}{\omega} \sigma_{tr}(\mathbf{q}, \omega, \nu). \quad (2.2)$$

Taking into account (2.1) and equality (2.2) we will write expression for the transversal dielectric permeability

$$\varepsilon_{tr}(\mathbf{q}, \omega, \bar{\nu}) = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + \frac{\hbar^2}{8\pi^3 m N} \int \Xi(\mathbf{k}, \mathbf{q})(f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}}) \mathbf{k}_{\perp}^2 d\mathbf{k} \right]. \quad (2.3)$$

Here  $\omega_p$  is the plasma (Langmuir) frequency,  $\omega_p^2 = 4\pi e^2 N/m$ .

Let's notice, that the kernel from subintegral expression from (2.1) can be we can present in the form of decomposition on partial fractions

$$\begin{aligned} \Xi(\mathbf{k}, \mathbf{q}) &\equiv \frac{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}} - i\hbar\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})}{(\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}})\{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}} - \hbar[\omega + i\hbar\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})]\}} = \\ &= \frac{i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})}{\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})} \cdot \frac{1}{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}}} + \\ &+ \frac{\omega}{\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})} \cdot \frac{1}{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}} - \hbar[\omega + i\hbar\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})]}. \end{aligned}$$

Hence, for transversal electric conductivity and dielectric permeability we have explicit representations

$$\sigma_{tr}(\mathbf{q}, \omega, \bar{\nu}) = \frac{ie^2 N}{m\omega} \left[ 1 + \frac{\hbar^2}{8\pi^3 m N} \int \frac{i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})}{\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})} \cdot \frac{f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}}}{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}}} \mathbf{k}_{\perp}^2 d\mathbf{k} + \right. \\ \left. + \frac{\hbar^2 \omega}{8\pi^3 m N} \int \frac{1}{\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})} \cdot \frac{(f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}}) \mathbf{k}_{\perp}^2 d\mathbf{k}}{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}} - \hbar[\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})]} \right], \quad (2.4)$$

and

$$\varepsilon_{tr}(\mathbf{q}, \omega, \bar{\nu}) = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + \frac{\hbar^2}{8\pi^3 m N} \int \frac{i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})}{\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})} \cdot \frac{f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}}}{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}}} \mathbf{k}_{\perp}^2 d\mathbf{k} + \right. \\ \left. + \frac{\hbar^2 \omega}{8\pi^3 m N} \int \frac{1}{\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})} \cdot \frac{(f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}}) \mathbf{k}_{\perp}^2 d\mathbf{k}}{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}} - \hbar[\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})]} \right]. \quad (2.5)$$

If to enter designations

$$J_{\bar{\nu}} = \frac{\hbar^2}{8\pi^3 m N} \int \frac{i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})}{\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})} \cdot \frac{f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}}}{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}}} \mathbf{k}_{\perp}^2 d\mathbf{k}$$

and

$$J_{\omega} = \frac{\hbar^2 \omega}{8\pi^3 m N} \int \frac{1}{\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})} \cdot \frac{(f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}}) \mathbf{k}_{\perp}^2 d\mathbf{k}}{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}} - \hbar[\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})]},$$

then expression (2.4) for electric conductivity and (2.5) for dielectric permeability will be transformed to the following form

$$\sigma_{tr}(\mathbf{q}, \omega, \bar{\nu}) = \frac{ie^2 N}{m\omega} (1 + J_{\bar{\nu}} + J_{\omega}) \quad (2.6)$$

and

$$\varepsilon_{tr}(\mathbf{q}, \omega, \bar{\nu}) = 1 - \frac{\omega_p^2}{\omega^2} (1 + J_{\bar{\nu}} + J_{\omega}). \quad (2.7)$$

Integrals  $J_{\bar{\nu}}$  and  $J_{\omega}$  we can transform to the following form

$$J_{\bar{\nu}} = \frac{i\hbar^2}{8\pi^3 m N} \int \left[ \frac{\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})}{[\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})] \{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}}\}} - \right. \\ \left. - \frac{\bar{\nu}(\mathbf{k} + \mathbf{q}, \mathbf{k})}{[\omega + i\bar{\nu}(\mathbf{k} + \mathbf{q}, \mathbf{k})] \{\mathcal{E}_{\mathbf{k}+\mathbf{q}} - \mathcal{E}_{\mathbf{k}}\}} \right] f_{\mathbf{k}} \mathbf{k}_{\perp}^2 d\mathbf{k} \quad (2.8)$$

and

$$J_\omega = \frac{\omega \hbar^2}{8\pi^3 m N} \int \left[ \frac{1}{[\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})]\{\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}-\mathbf{q}} - \hbar[\omega + i\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})]\}} - \frac{1}{[\omega + i\bar{\nu}(\mathbf{k} + \mathbf{q}, \mathbf{k})]\{\mathcal{E}_{\mathbf{k}+\mathbf{q}} - \mathcal{E}_{\mathbf{k}} - \hbar[\omega + i\bar{\nu}(\mathbf{k} + \mathbf{q}, \mathbf{k})]\}} \right] f_{\mathbf{k}} \mathbf{k}_\perp^2 d\mathbf{k}. \quad (2.9)$$

### 3. Non-degenerate plasma

Instead of the vector  $\mathbf{k}$  we will enter the dimensionless vector  $\mathbf{K}$  by following equality  $\mathbf{K} = \frac{\mathbf{k}}{k_T}$ ,  $k_T = \frac{p_T}{\hbar}$ , where  $k_T$  is the thermal wave number,  $p_T = mv_T$  is the thermal electron momentum,

$$v_T = \frac{1}{\sqrt{\beta}} = \sqrt{\frac{2k_B T}{m}}$$

is the thermal electron velocity,  $k_B$  is the Boltzmann constant,  $T$  is the plasma temperature.

Then

$$(k^2 - k_x^2)d^3k = k_T^5(K^2 - K_x^2)d^3K = k_T^5 K_\perp^2 d^3K,$$

where

$$K_\perp^2 = K^2 - K_x^2 = K_y^2 + K_z^2.$$

Further we will consider the case of non-degenerate plasmas. Then we have

$$\left(\frac{mv_T}{\hbar}\right)^3 \equiv \left(\frac{p_T}{\hbar}\right)^3 \equiv k_T^3 = \frac{\pi^2}{f_2(\alpha)} N.$$

Here

$$f_2(\alpha) = \int_0^\infty f_F(K, \alpha) K^2 dK = \int_0^\infty \frac{K^2 dK}{1 + e^{K^2 - \alpha}},$$

where

$$f_F(K, \alpha) = \frac{1}{1 + e^{K^2 - \alpha}}.$$

Hence,

$$(k^2 - k_x^2)d^3k = \frac{\pi^2}{f_2(\alpha)} N \frac{m^2 v_T^2}{\hbar^2} (K^2 - K_x^2)d^3K.$$

Energy  $\mathcal{E}_{\mathbf{k}}$  we will express through thermal energy. We have

$$\mathcal{E}_{\mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m} = \frac{\hbar^2 k_T^2}{2m} \mathbf{K}^2 = \frac{p_T^2}{2m} \mathbf{K}^2 = \mathcal{E}_T \mathbf{K}^2 \equiv \mathcal{E}_{\mathbf{K}}.$$

Here  $\mathcal{E}_T = \frac{p_T^2}{2m}$  is the thermal electron energy.

Besides,

$$k_B T = \frac{2k_B T}{m} \cdot \frac{m}{2} = \frac{mv_T^2}{2} = \mathcal{E}_T,$$

$$\frac{\mathcal{E}_{\mathbf{k}}}{k_B T} = \mathbf{K}^2 = K^2, \quad K = |\mathbf{K}|.$$

Absolute Fermi–Dirac’s distribution  $f_{\mathbf{k}}$  for non-degenerate plasma is distribution

$$f_{\mathbf{k}} = \frac{1}{1 + \exp\left(\frac{\mathcal{E}_{\mathbf{k}}}{k_B T} - \alpha\right)} = \frac{1}{1 + e^{\mathbf{K}^2 - \alpha}} = f_{\mathbf{K}} \equiv f_F(K, \alpha).$$

Calculation of transversal electric conductivity can be spent on any of formulas (2.4) – (2.7).

In the same way we receive

$$\mathcal{E}_{\mathbf{k}-\mathbf{q}} = \frac{\hbar^2 (k_T \mathbf{K} - \mathbf{q})^2}{2m} = \frac{\hbar^2 k_T^2}{2m} \left( \mathbf{K} - \frac{\mathbf{q}}{k_T} \right)^2.$$

Further we introduce dimensionless wave vector  $\mathbf{Q} = \frac{\mathbf{q}}{k_T}$ . Then

$$\mathcal{E}_{\mathbf{k}-\mathbf{q}} = \frac{\hbar^2 k_T^2}{2m} \left( \mathbf{K} - \mathbf{Q} \right)^2 = \mathcal{E}_T (\mathbf{K} - \mathbf{Q})^2 = \mathcal{E}_{\mathbf{K}-\mathbf{Q}}.$$

We notice that

$$\begin{aligned} \mathcal{E}_{\mathbf{K}} - \mathcal{E}_{\mathbf{K}-\mathbf{Q}} &= \mathcal{E}_T \mathbf{K}^2 - \mathcal{E}_T (\mathbf{K} - \mathbf{Q})^2 = \mathcal{E}_T [2\mathbf{K} \cdot \mathbf{Q} - Q^2] = \\ &= 2Q \mathcal{E}_T \left( K_x - \frac{Q}{2} \right). \end{aligned}$$

Besides

$$\mathcal{E}_{\mathbf{K}} - \mathcal{E}_{\mathbf{K}-\mathbf{Q}} - \hbar[\omega + i\bar{\nu}(\mathbf{K}, \mathbf{K} - \mathbf{Q})] = 2\mathcal{E}_T Q \left( K_x - \frac{z^-}{Q} - \frac{Q}{2} \right),$$

$$\mathcal{E}_{\mathbf{K}+\mathbf{Q}} - \mathcal{E}_{\mathbf{K}} - \hbar[\omega + i\bar{\nu}(\mathbf{K} + \mathbf{Q}, \mathbf{K})] = 2\mathcal{E}_T Q \left( K_x - \frac{z^+}{Q} + \frac{Q}{2} \right),$$

where

$$z^\pm = x + iy^\pm, \quad x = \frac{\omega}{k_T v_T},$$

$$y^- = \frac{\bar{\nu}(\mathbf{K}, \mathbf{K} - \mathbf{Q})}{k_T v_T}, \quad y^+ = \frac{\bar{\nu}(\mathbf{K} + \mathbf{Q}, \mathbf{K})}{k_T v_T}.$$

Now formulas (2.4) and (2.5) we can rewrite in the form

$$\sigma_{tr}(Q, x, y) = \frac{ie^2 N}{m\omega} \left(1 + J_\omega + J_{\bar{\nu}}\right) \quad (3.1)$$

and

$$\varepsilon_{tr}(Q, x, y) = 1 - \frac{x_p^2}{x^2} \left(1 + J_\omega + J_{\bar{\nu}}\right), \quad x_p = \frac{\omega_p}{k_T v_T}. \quad (3.2)$$

In formulas (3.1) and (3.2) there are the following designations

$$J_\omega = J_\omega(Q, x, y) = \frac{1}{8\pi f_2(\alpha)Q} \int \left[ \frac{x}{(x + iy^-)(K_x - z^-/Q - Q/2)} - \frac{x}{(x + iy^+)(K_x - z^+/Q + Q/2)} \right] f_{\mathbf{K}} \mathbf{K}_\perp^2 d^3 K,$$

$$J_{\bar{\nu}}(Q, x, y) = \frac{1}{8\pi f_2(\alpha)Q} \int \left[ \frac{iy^-}{(x + iy^-)(K_x - Q/2)} - \frac{iy^+}{(x + iy^+)(K_x + Q/2)} \right] f_{\mathbf{K}} \mathbf{K}_\perp^2 d^3 K.$$

#### 4. Frequency of collisions is proportional to the module of a wave vector

Let's consider the special case, when frequency of collisions is proportional to the module of a wave vector

$$\nu(\mathbf{k}) = \nu_0 |\mathbf{k}|.$$

Then

$$\bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q}) = \frac{\nu(\mathbf{k}) + \nu(\mathbf{k} - \mathbf{q})}{2} = \frac{\nu_0}{2} (|\mathbf{k}| + |\mathbf{k} - \mathbf{q}|),$$

and

$$\bar{\nu}(\mathbf{k} + \mathbf{q}, \mathbf{k}) = \frac{\nu(\mathbf{k} + \mathbf{q}) + \nu(\mathbf{k})}{2} = \frac{\nu_0}{2} (|\mathbf{k} + \mathbf{q}| + |\mathbf{k}|).$$

The quantity  $\nu_0$  we take in the form  $\nu_0 = \frac{\nu}{k_T}$ , where  $k_T$  is the thermal wave number,  $k_T = \frac{mv_T}{\hbar}$ ,  $\hbar$  is the Planck's constant,  $v_T$  is the thermal electron velocity. Now we have

$$\nu(\mathbf{k}) = \frac{\nu}{k_T} |\mathbf{k}|. \quad (4.1)$$

Let's notice, that at  $k = k_T$ :  $\nu(k_T) = \nu$ . So, further in previous formulas frequency collisions according to (4.1) it is equal

$$\begin{aligned} \bar{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q}) &= \frac{\nu}{2k_T} (|\mathbf{k}| + |\mathbf{k} - \mathbf{q}|) = \frac{\nu}{2} (|\mathbf{K}| + |\mathbf{K} - \mathbf{Q}|) = \bar{\nu}(\mathbf{K}, \mathbf{K} - \mathbf{Q}), \\ \bar{\nu}(\mathbf{k} + \mathbf{q}, \mathbf{k}) &= \frac{\nu}{2k_T} (|\mathbf{k} + \mathbf{q}| + |\mathbf{k}|) = \frac{\nu}{2} (|\mathbf{K} + \mathbf{Q}| + |\mathbf{K}|) = \bar{\nu}(\mathbf{K} + \mathbf{Q}, \mathbf{K}). \end{aligned}$$

Hence, quantities  $z^\pm$  are equal

$$z^\pm = x + iy\rho^\pm, \quad y = \frac{\nu}{k_F v_F},$$

$$\rho^\pm = \frac{1}{2} \left( \sqrt{K_x^2 + K_y^2 + K_z^2} + \sqrt{(K_x \pm Q)^2 + K_y^2 + K_z^2} \right).$$

Now integrals  $J_{\bar{\nu}}$  and  $J_\omega$  are accordingly equal

$$\begin{aligned} J_{\bar{\nu}} &= \frac{iy}{8\pi Q f_2(\alpha)} \int \left( \frac{\rho^-}{(x + iy\rho^-)(K_x - Q/2)} - \right. \\ &\quad \left. - \frac{\rho^+}{(x + iy\rho^+)(K_x + Q/2)} \right) f_{\mathbf{K}} \mathbf{K}_\perp^2 d^3 K, \end{aligned} \quad (4.2)$$

and

$$\begin{aligned} J_\omega &= \frac{x}{8\pi Q f_2(\alpha)} \int \left( \frac{1}{(x + iy\rho^-)(K_x - z^-/Q - Q/2)} - \right. \\ &\quad \left. - \frac{1}{(x + iy\rho^+)(K_x - z^+/Q + Q/2)} \right) f_{\mathbf{K}} \mathbf{K}_\perp^2 d^3 K. \end{aligned} \quad (4.3)$$

Three-dimensional integrals (4.2) and (4.3) after passing to polar coordinates in a plane  $(K_y, K_z)$  are easily reduced to the double

$$J_{\bar{\nu}} = \frac{iy}{4Q f_2(\alpha)} \int_{-\infty}^{+\infty} dK_x \int_0^\infty \left( \frac{\rho^-}{(x + iy\rho^-)(K_x - Q/2)} - \right.$$



$$-\frac{\rho^+}{(x+iy\rho^+)(K_x+Q/2)}\bigg)f_F(K_x, r, \alpha)r^3dr, \quad (4.4)$$

and

$$J_\omega = \frac{x}{4Qf_2(\alpha)} \int_{-\infty}^{+\infty} dK_x \int_0^\infty \left( \frac{1}{(x+iy\rho^-)(K_x-z^-/Q-Q/2)} - \frac{1}{(x+iy\rho^+)(K_x-z^+/Q+Q/2)} \right) f_F(K_x, r, \alpha)r^3dr. \quad (4.5)$$

Here

$$f_F(K_x, r, \alpha) = \frac{1}{1 + e^{K_x^2 + r^2 - \alpha}} = f_F(K, \alpha), \quad r^2 = K_y^2 + K_z^2,$$

$$\rho^\pm = \rho^\pm(K_x, r) = \frac{1}{2} \left( \sqrt{K_x^2 + r^2} + \sqrt{(K_x \pm Q)^2 + r^2} \right).$$

Let's notice, that in case of constant frequency of collisions  $\rho^\pm = 1$  and formulas (4.4) and (4.5) pass in the following

$$J_\omega = \frac{x}{4(x+iy)f_2(\alpha)} I(Q, z),$$

where

$$I(Q, z) = \int_{-\infty}^{\infty} \frac{f_3(K_x, \alpha) dK_x}{(K_x - z/Q)^2 - (Q/2)^2},$$

$$f_3(K_x, \alpha) = \int_0^\infty \ln(1 + e^{K_x^2 + r^2 - \alpha}) r dr,$$

$$J_\nu = \frac{iy}{4(x+iy)} I(Q, 0), \quad z = x + iy.$$

By means of these expressions we receive known formulas for electric conductivity and dielectric permeability of quantum collisional degenerate plasmas with constant frequency of collisions of particles [7]

$$\frac{\sigma_{tr}}{\sigma_0} = \frac{iy}{x} \left[ 1 + \frac{1}{4f_2(\alpha)} \frac{xI(Q, z) + iyI(Q, 0)}{x + iy} \right] \quad (4.6)$$

and

$$\varepsilon_{tr} = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + \frac{1}{4f_2(\alpha)} \frac{xI(Q, z) + iyI(Q, 0)}{x + iy} \right]. \quad (4.7)$$

## 5. Maxwellian plasma

Dimensionless vectors  $\mathbf{K}$  and  $\mathbf{Q}$  are entered as well as in the case of non-degenerate plasmas, however now

$$f_{\mathbf{k}} = 4\pi^{3/2} \frac{N}{k_T^3} \exp\left(-\frac{\mathcal{E}_{\mathbf{k}}}{\mathcal{E}_T}\right) = 4\pi^{3/2} \frac{N}{k_T^3} e^{-\mathbf{K}^2} \equiv f_{\mathbf{K}},$$

thus

$$N \equiv f_{\mathbf{k}} \frac{2d^3\mathbf{p}}{(2\pi\hbar)^3}, \quad \mathbf{k}_{\perp}^2 d\mathbf{k} = k_T^5 \mathbf{K}_{\perp}^2 d^3K.$$

Electric conductivity and dielectric permeability is calculated again under formulas (3.1) and (3.2)

$$\sigma_{tr}(Q, x, y) = \frac{ie^2 N}{m\omega} \left(1 + J_{\omega} + J_{\bar{\nu}}\right)$$

and

$$\varepsilon_{tr}(Q, x, y) = 1 - \frac{x_p^2}{x^2} \left(1 + J_{\omega} + J_{\bar{\nu}}\right)$$

In these formulas

$$\begin{aligned} J_{\omega} = \frac{x}{2\pi^{3/2}Q} \int & \left( \frac{1}{(x + iy\rho^-)(K_x - z^-/Q - Q/2)} - \right. \\ & \left. - \frac{1}{(x + iy\rho^+)(K_x - z^+/Q + Q/2)} \right) f_{\mathbf{K}} \mathbf{K}_{\perp}^2 d^3K, \\ J_{\bar{\nu}} = \frac{iy}{2\pi^{3/2}Q} \int & \left( \frac{\rho^-}{(x + iy\rho^-)(K_x - Q/2)} - \right. \\ & \left. - \frac{\rho^+}{(x + iy\rho^+)(K_x + Q/2)} \right) f_{\mathbf{K}} \mathbf{K}_{\perp}^2 d^3K. \end{aligned}$$

After integration in the plane  $(K_y, K_z)$  previous formulas have the following form

$$\begin{aligned} J_{\omega} = \frac{x}{\sqrt{\pi}Q} \int_{-\infty}^{+\infty} e^{-K_x^2} dK_x \int_0^{\infty} & \left( \frac{1}{(x + iy\rho^-)(K_x - z^-/Q - Q/2)} - \right. \\ & \left. - \frac{1}{(x + iy\rho^+)(K_x - z^+/Q + Q/2)} \right) e^{-r^2} r^3 dr, \end{aligned}$$

$$J_{\bar{\nu}} = \frac{iy}{\sqrt{\pi}Q} \int_{-\infty}^{+\infty} e^{-K_x^2} dK_x \int_0^{\infty} \left( \frac{\rho^-}{(x + iy\rho^-)(K_x - Q/2)} - \frac{\rho^+}{(x + iy\rho^+)(K_x + Q/2)} \right) e^{-r^2} r^3 dr.$$

In case of constant frequency of collisions we have  $\rho^{\pm} = 1$ , thus

$$J_{\omega} = \frac{x}{x + iy} I(Q, z), \quad I(Q, z) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-K_x^2} dK_x}{(K_x - z/Q)^2 - (Q/2)^2},$$

$$J_{\nu} = \frac{iy}{x + iy} I(Q, 0), \quad I(Q, 0) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-K_x^2} dK_x}{K_x^2 - (Q/2)^2}.$$

To these formulas it is possible to come and other method. Really, in formulas (3.6) and (3.7) we will pass to the limit at  $\alpha \rightarrow -\infty$ . For this purpose we will calculate the following limit

$$\lim_{\alpha \rightarrow -\infty} \frac{f_3(K_x, \alpha)}{f_2(\alpha)} = \frac{2}{\sqrt{\pi}} e^{-K_x^2}. \quad (5.1)$$

By means of equality (5.1) from previous formulas (3.6) and (3.7) we receive known formulas for quantum maxwellian plasmas with constant frequency of collisions

$$\sigma_{tr}(x, y, Q) = i\sigma_0 \frac{y}{x} \left[ 1 + \frac{xI(Q, z) + iyI(Q, 0)}{x + iy} \right]$$

and

$$\varepsilon_{tr}(x, y, Q) = 1 - \frac{x_p^2}{x^2} \left[ 1 + \frac{xI(Q, z) + iyI(Q, 0)}{x + iy} \right].$$

Here

$$I(Q, z) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-K_x^2} dK_x}{(K_x - z/Q)^2 - (Q/2)^2}.$$

On Figs. 1 – 12 we will present comparison real and imaginary parts of dielectric function,  $x_p = 1$ . Figures 1-6 are devoted to maxwellian plasma, and figures 7-12 are devoted to non-degenerate plasma.

## 5. Conclusion

In the present work formulas for the transversal electric conductivity and dielectric permeability into quantum non-degenerate and maxwellian collisional plasma are deduced. Frequency of collisions of particles depends arbitrarily on a wave vector. The special case of frequency of collisions proportional to the module of a wave vector is considered. The graphic analysis of the real and imaginary parts of dielectric function is made.

## REFERENCES

- [1] *Klimontovich Y. and Silin V. P.* The Spectra of Systems of Interacting Particles// JETP (Journal Experimental Theoreticheskoi Fiziki), **23**, 151 (1952).
- [2] *Lindhard J.* On the properties of a gas of charged particles// Kongelige Danske Videnskabernes Selskab, Matematisk–Fysiske Meddelelser. V. 28, No. 8 (1954), 1–57.
- [3] *Kliwer K. L., Fuchs R.* Lindhard Dielectric Functions with a Finite Electron Lifetime// Phys. Rev. 1969. V. 181. No. 2. P. 552–558.
- [4] *Mermin N. D.* Lindhard Dielectric Functions in the Relaxation–Time Approximation. Phys. Rev. B. 1970. V. 1, No. 5. P. 2362–2363.
- [5] *Latyshev A.V., Yushkanov A.A.* Longitudinal permittivity of a quantum degenerate collisional plasma// Teor. and Mathem. Physics, **169**(3): 1739–1749 (2011).
- [6] *Latyshev A.V., Yushkanov A.A.* Transverse Electric Conductivity in Collisional Quantum Plasma// Plasma Physics Reports, 2012, Vol. 38, No. 11, pp. 899–908.
- [7] *Latyshev A.V., Yushkanov A.A.* Transverse electric conductivity in quantum collisional plasma in Mermin approach// arXiv:1109.6554v1 [math-ph] 29 Sep 2011.

- [8] *Latyshev A.V., Yushmanov A.A.* Longitudinal electric conductivity and dielectric permeability in quantum plasma with variable frequency of collisions in Mermin' approach// arXiv:1212.5659v1 [physics.plasma-ph] 17 Jan 2013, 28 p.
- [9] *Latyshev A.V., Yushmanov A.A.* Transverse and Longitudinal Permittivities of a Gaseous Plasma with an Electron Collision Frequency Proportional to the Electron Velocity. – Plasma Physics Report, 2007, Vol. 33, No. 8, pp. 696–702 (Fizika Plasmy, Vol. 33, No. 8, pp. 762–768, russian).
- [10] *Latyshev A.V., Yushmanov A.A.* Skin Effect in a Gaseous Plasma with a Collision Frequency Proportional to the Electron Velocity. – Plasma Physics Report. 2006. Vol. 32. No. 11, pp. 943 – 948 (Fizika Plasmy, Vol. 32, No. 11, pp. 1021–1026, russian).
- [11] *Latyshev A.V., Yushmanov A.A.* Longitudinal dielectric permeability into quantum degenerate plasma with frequency of collisions proportional to the module of a wave vector// arXiv:1302.0718v1 [physics.plasma-ph] 4 Feb 2013, 20 p.
- [12] *Latyshev A.V., Yushmanov A.A.* Longitudinal dielectric permeability into quantum non-degenerate and maxwellian plasma with frequency of collisions proportional to the module of a wave vector// arXiv:1302.0838v1 [physics.plasma-ph] 4 Feb 2013, 22 p.
- [13] *Latyshev A.V., Yushmanov A.A.* Transverse electric conductivity and dielectric permeability in quantum degenerate collisional plasma with variable collision frequency in Mermin's approach// arXiv:1302.3237v1 [physics.plasma-ph], 13 Feb 2013, 22 p.
- [14] *Manfredi G.* How to model quantum plasmas// arXiv: quant - ph/0505004.
- [15] *Anderson D., Hall B., Lisak M., and Marklund M.* Statistical effects in the multistream model for quantum plasmas// Phys. Rev. E **65** (2002), 046417.
- [16] *Andrés P., de Monreal R., and Flores F.* Relaxation–time effects in the transverse dielectric function and the electromagnetic properties

- of metallic surfaces and small particles// Phys. Rev. **B**. 1986. Vol. 34, No. 10, 7365–7366.
- [17] *Shukla P. K. and Eliasson B.* Nonlinear aspects of quantum plasma physics// Uspekhy Fiz. Nauk, **53**(1) 2010;[V. 180. No. 1, 55-82 (2010) (in Russian)].
  - [18] *Eliasson B. and Shukla P.K.* Dispersion properties of electrostatic oscillations in quantum plasmas// arXiv:0911.4594v1 [physics.plasm-ph] 24 Nov 2009, 9 pp.
  - [19] *Opher M., Morales G. J., Leboeuf J. N.* Krook collisional models of the kinetic susceptibility of plasmas// Phys. Rev. E. V.66, 016407, 2002.
  - [20] *Gelder van, A.P.* Quantum Corrections in the Theory of the Anomalous Skin Effect// Phys. Rev. 1969. Vol. 187. No. 3. P. 833–842.
  - [21] *Fuchs R., Kliewer K. L.* Surface plasmon in a semi-infinite free-electron gas// Phys. Rev. B. 1971. V. 3. No. 7. P. 2270–2278.
  - [22] *Fuchs R., Kliewer K. L.* Optical properties of an electron gas: further studies of a nonlocal description// Phys. Rev. 1969. V. 185. No. 3. P. 905–913.
  - [23] *Dressel M., Grüner G.* Electrodynamics of Solids. Optical Properties of Electrons in Matter. - Cambridge. Univ. Press. 2003. 487 p.
  - [24] *Wierling A.* Interpolation between local field corrections and the Drude model by a generalized Mermin approach// arXiv:0812.3835v1 [physics.plasm-ph] 19 Dec 2008.
  - [25] *Brodin G., Marklund M., Manfredi G.* Quantum Plasma Effects in the Classical Regime// Phys. Rev. Letters. **100**, (2008). P. 175001-1–175001-4.
  - [26] *Manfredi G. and Haas F.* Self-consistent fluid model for a quantum electron gas// Phys. Rev. B **64** (2001), 075316.
  - [27] *Reinholz H., Röpke G.* Dielectric function beyond the random-phase approximation: Kinetic theory versus linear response theory// Phys. Rev., **E 85**, 036401 (2012).

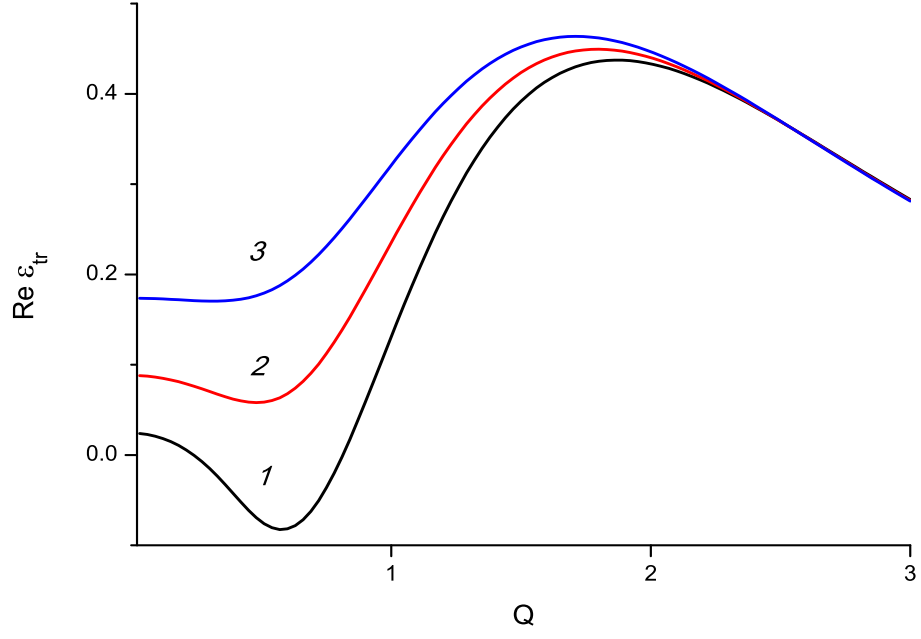


Fig. 1. Real part of dielectric function,  $x = 1$ . Curves 1,2,3 correspond to values  $y = 0.1, 0.2, 0.3$ .

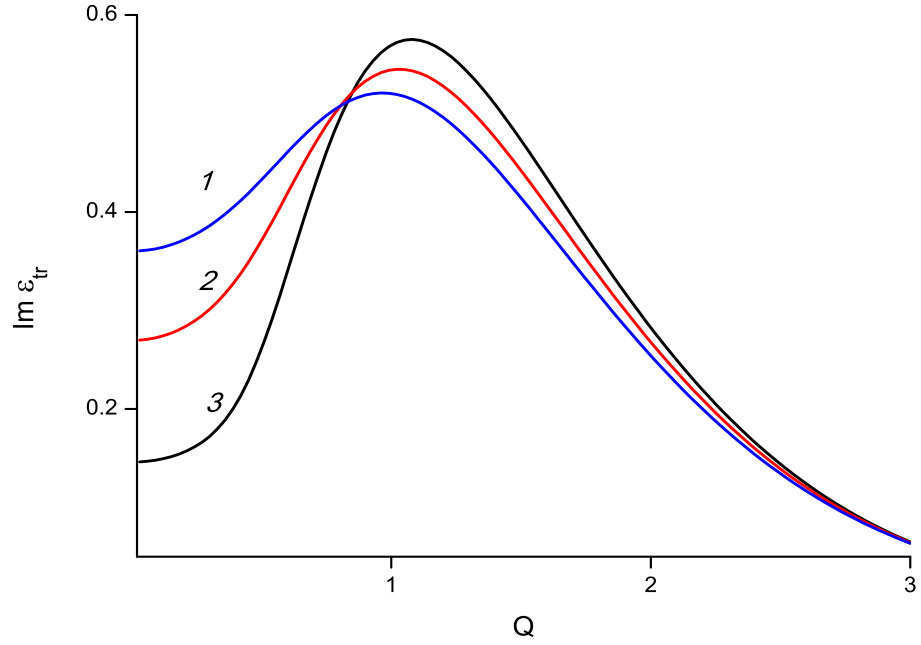


Fig. 2. Imaginare part of dielectric function,  $x = 1$ . Curves 1,2,3 correspond to values  $y = 0.1, 0.2, 0.3$ .

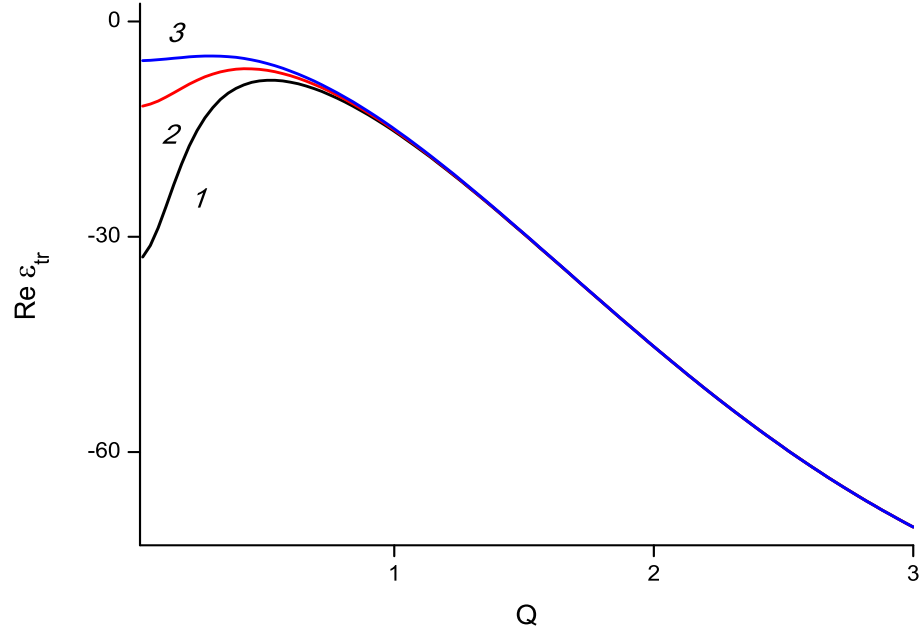


Fig. 3. Real part of dielectric function,  $x = 0.1$ . Curves 1,2,3 correspond to values  $y = 0.1, 0.2, 0.3$ .

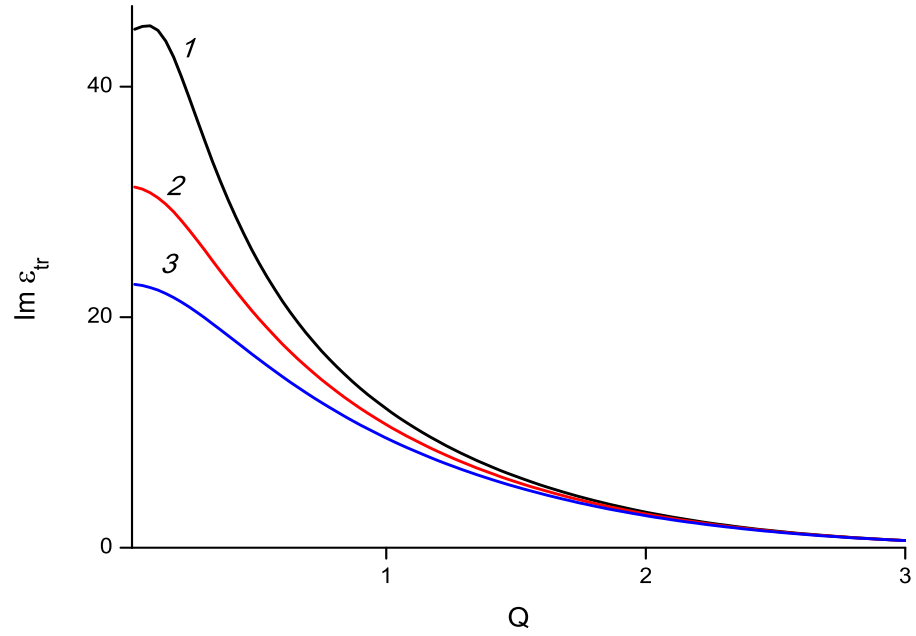


Fig. 4. Imaginare part of dielectric function,  $x = 0.1$ . Curves 1,2,3 correspond to values  $y = 0.1, 0.2, 0.3$ .



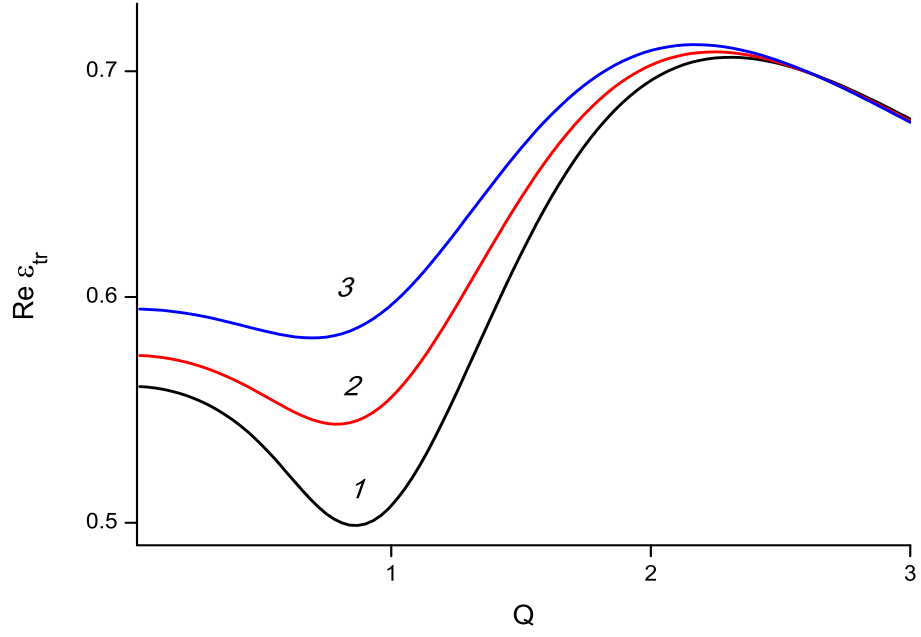


Fig. 5. Real part of dielectric function,  $x = 1.5$ . Curves 1,2,3 correspond to values  $y = 0.1, 0.2, 0.3$ .

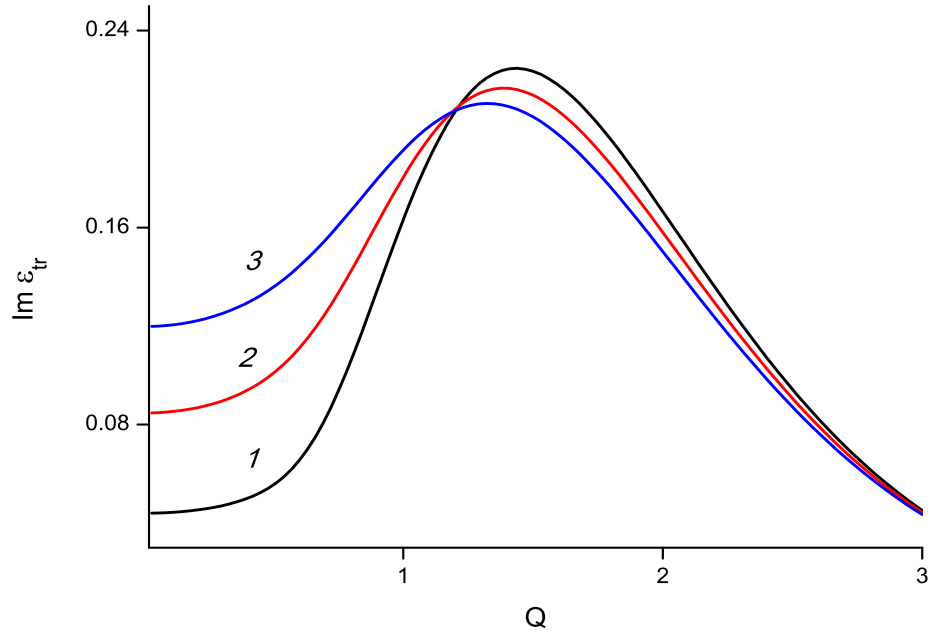


Fig. 6. Imaginare part of dielectric function,  $x = 1.5$ . Curves 1,2,3 correspond to values  $y = 0.1, 0.2, 0.3$ .

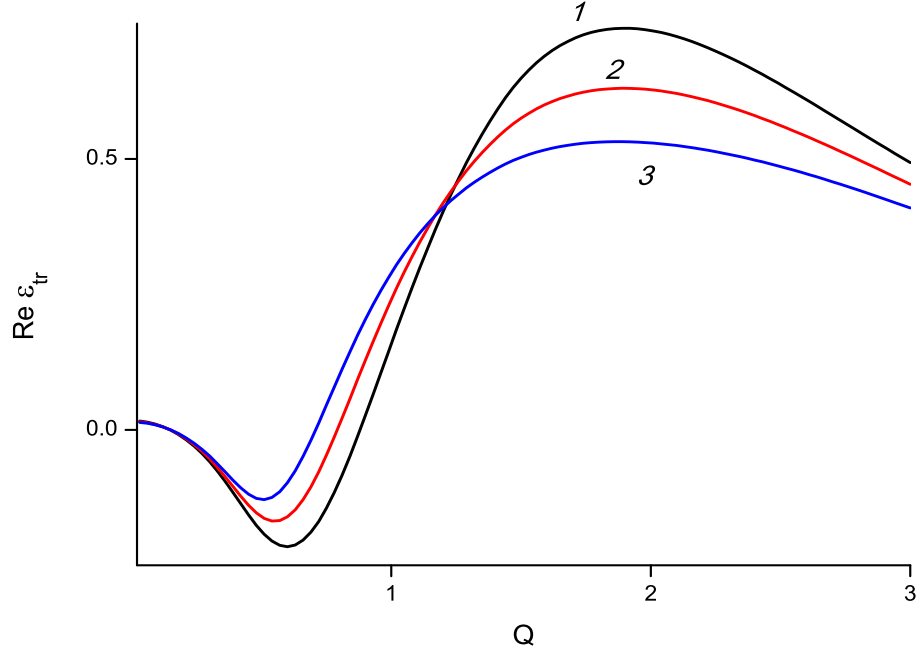


Fig. 7. Real part of dielectric function,  $x = 1, y = 0.1$ . Curves 1,2,3 correspond to values  $\alpha = -2, 0, +1$ .

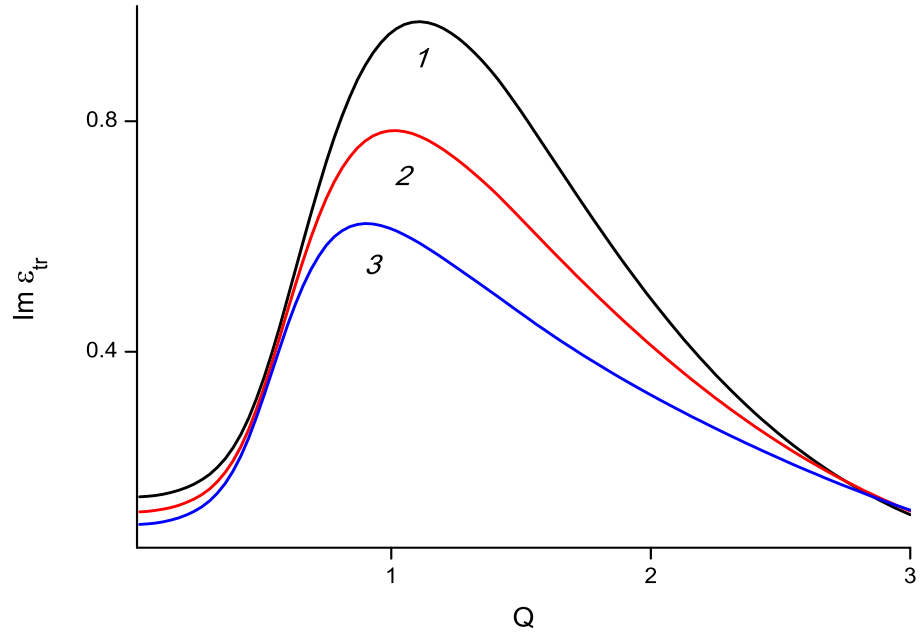


Fig. 8. Imaginare part of dielectric function,  $x = 1, y = 0.1$ . Curves 1,2,3 correspond to values  $\alpha = -2, 0, +1$ .

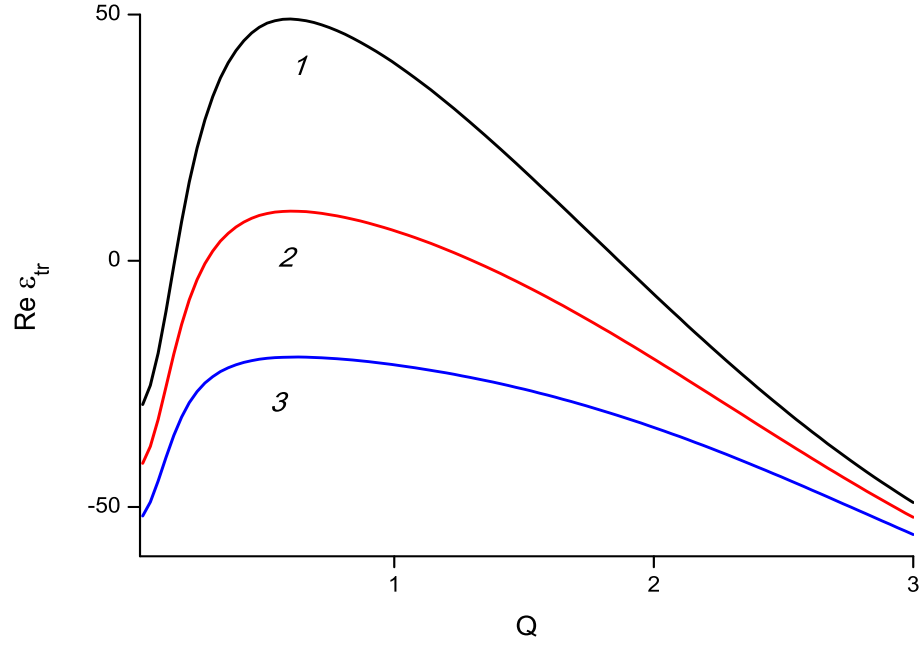


Fig. 9. Real part of dielectric function,  $x = 0.1, y = 0.1$ . Curves 1,2,3 correspond to values  $\alpha = -2, 0, +1$ .

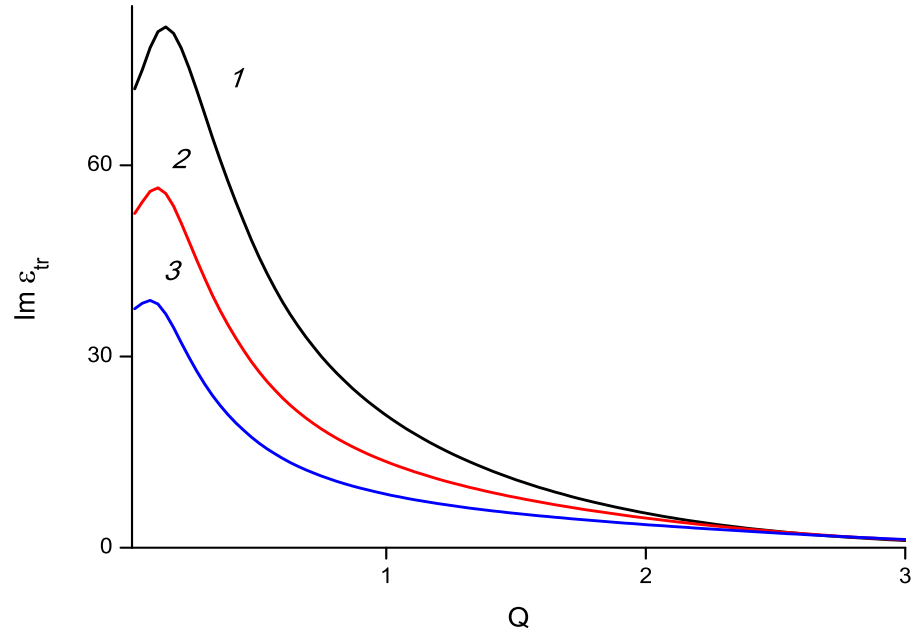


Fig. 10. Imaginare part of dielectric function,  $x = 0.1$ . Curves 1,2,3 correspond to values  $\alpha = -2, 0, +1$ .

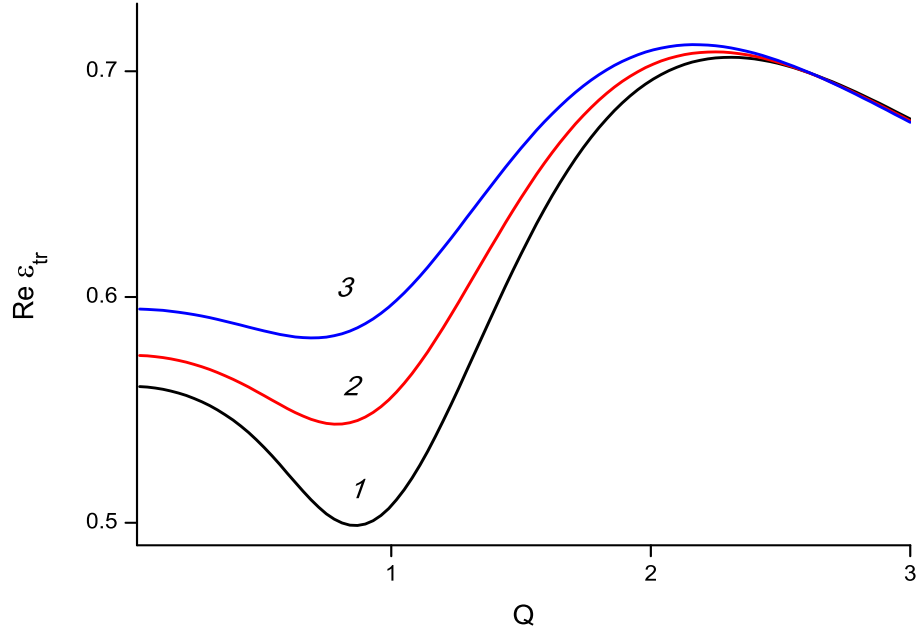


Fig. 11. Real part of dielectric function,  $x = 1.5, y = 0.1$ . Curves 1,2,3 correspond to values  $\alpha = -2, 0, +1$ .

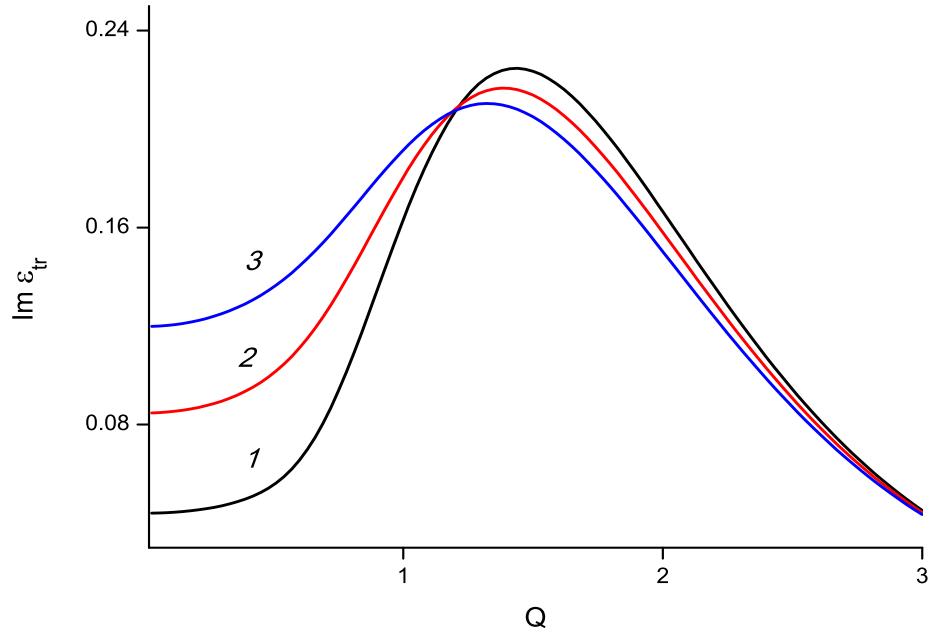


Fig. 12. Imaginare part of dielectric function,  $x = 1.5, y = 0.1$ . Curves 1,2,3 correspond to values  $\alpha = -2, 0, +1$ .

